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(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2014

FIRST YEAR

MATHEMATICS (General)

Paper : II

Date : 26/05/2014

Time : 11 am – 2 pm

Full Marks : 75

(Use a separate Answer Book for each group)

Group – A

Answer **any three** questions : [3×5]

1. a) Find the co-ordinates of the point where the origin is to be shifted to eliminate the first degree terms of the equation $3x^2+8xy+3y^2-2x+2y-2=0$. [2]

   b) If the expression ax + by changes to $a'x'+b'y'$ by a rotation of rectangular axes about the origin, prove that $a^2+b^2=a'^2+b'^2$. [3]

2. a) State the conditions under which the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of intersecting straight lines. [2]

   b) Find the equation of the pair of bisectors of the angles between the pair of straight lines given by $2x^2+3xy+5y^2=0$. Also find the angle between the given straight lines. [2+1]

3. a) Find the equation of the chord of contacts for tangents from an external point to the circle $x^2+y^2+2gx+2fy+c=0$. [3]

   b) Write the equation of the pair of tangents from (12,13) to the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$. [2]

4. a) Define polar of a point with respect to a conic. [1]

   b) Show that the locus of the poles of the tangents to the parabola $y^2=4bx$ with respect to the parabola $y^2=4ax$ is another parabola. [4]

5. a) If PSP' be a focal chord of the conic $\frac{\ell}{r}=1+e\cos\theta$ then show that $\frac{1}{SP}+\frac{1}{SP'}=\frac{2}{\ell}$. [3]

   b) Show that the straight line $\frac{\ell}{r}=A\cos\theta+B\sin\theta$ touches the conic $\frac{\ell}{r}=1+e\cos\theta$ if $(A-e)^2+B^2=1$. [2]

Group – B

Answer **any three** questions : [3×5]

6. a) Determine the values of λ and μ for which the vectors $-3\vec{i}+4\vec{j}+\lambda\vec{k}$ and $\mu\vec{i}+8\vec{j}+6\vec{k}$ are collinear. [3]

   b) Find the vector equation of a plane passing through the origin and parallel to the vectors $\vec{i}+2\vec{j}+3\vec{k}$ and $2\vec{i}-\vec{j}-4\vec{k}$. [2]

7. If $\vec{x},\vec{y},\vec{z}$ be three non-coplanar vectors then show that : $[\vec{x}\times\vec{y},\vec{y}\times\vec{z},\vec{z}\times\vec{x}]=[\vec{x}\,\vec{y}\,\vec{z}]^2$. [5]

8. Under what condition does the relation $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$ hold where $\vec{a},\vec{b},\vec{c}$ are given proper vectors? [5]

9. a) Find the torque about the point A(1,2,3) of a force of magnitude 5 units acting through the point (3,4,5) in the direction of the vector $2\vec{i}+3\vec{j}+4\vec{k}$. [3]

   b) A particle acted on by constant forces $3\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}-2\hat{j}+8\hat{k}$, is displaced from the origin to the point $2\hat{i}+\hat{j}+2\hat{k}$. Find the total work done by the forces. [2]

10. Show by vector method, that the perpendicular from the vertices of a triangle to the opposite sides are concurrent. [5]

Group – C

Answer **any five** questions : [5×5]

11. a) Prove that a convergent sequence of real numbers can't have more than one limit. [3]

b) If $\sum_{n=1}^{\infty} a_n$ is a convergent series of real numbers then show that $\lim_{n\to\infty} a_n = 0$. [2]

12. a) Test the convergence of the series : $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\ldots+\frac{n}{2^n}+\ldots$ [3]

b) State Leibnitz's test for alternating series. [2]

13. State and prove Lagrange's mean value theorem. [5]

14. Show that for all real x, $e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$ [5]

15. Find the point upon the plane $ax+by+cz=p$ at which the function $f(x,y,z)=x^2+y^2+z^2$ has the minimum value and find the minimum. Using the Lagrange's multiplier method. [5]

16. a) Prove that $\lim_{x\to\infty}\left\{x-\sqrt{(x-a)(x-b)}\right\}=\frac{a+b}{2}$. [1]

b) A rectangular box, open at the top, with given capacity V. Find the dimension of the box requiring least material for its construction. [4]

17. Find the asymptotes of $x^3+x^2y-xy^2-y^3+x^2-y^2=2$. [5]

18. a) Find the envelope of the family of straight lines $\frac{x}{a}+\frac{y}{b}=1$ where the parameters a,b are connected by the relation $ab=c^2$ (c being a constant). [3]

b) Examine the curve $(x^2+y^2)x-2y^2=0$ for singular points at the origin. [2]

Group – D

Answer **any one** question : [1×10]

19. a) Evaluate $\int_0^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx$. [4]

b) If $I_n=\int_0^{\pi/2}x^n \sin x\, dx\ (n>1)$, show that $I_n+n(n-1)I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$. [3]

c) Evaluate $\int\frac{2x^2+1}{(3x-1)(x+1)}dx$. [3]

20. a) Use Wallis' method to evaluate $\int_a^b\frac{1}{x^2}dx$,0< a < b. [4]

b) Evaluate $\int\frac{2\sin x+3\cos x}{3\sin x+4\cos x}dx$. [3]

c) Find the value of $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{(n^2+r^2)^{3/2}}$. [3]

Group – E

Answer **any one** question : [1×10]

21. a) Find the Integrating factors of the following differential equation and hence solve it :

$(3x+2y^2)y\,dx+2x(2x+3y^2)dy=0$. [5]

b) Reduce the following differential equation to a linear equation and hence solve it :

$(x^2y^3+2xy)dy=dx$. [1+4]

22. a) Solve : $x\dfrac{dy}{dx}+\sin 2y=x^4\cos^2 y$. [4]

b) Find the general solution as well as the singular solution of the differential equation :

$y=2px+p^4x^2$. [3+1]

c) Form the differential equation whose solution is $xy=Ae^x+Be^{-x}+x^2$ where A, B are parameters. [2]